

B.Sc IVth Sem

Assignment- Abstract Algebra

Last date of Submission: 15/03/2018

MM: 10

Q1. Consider three axioms of group: $G_1 =$ Associative, $G_2 =$ Existence of identity & $G_3 =$ Existence of inverse. It was defined in the order $G_1G_2G_3$. What are the possible orders that are acceptable for a definition? Which orders are not acceptable and why?

Q2. The following “definitions” of a group are taken as it is, including spelling and punctuation, from papers of students who wrote a bit too quickly and carelessly. Criticize them.

(a) A group is a set G such that

The operation on G is associative

There is an identity element (e) in G

For every $a \in G$, there is an a' (inverse of each element)

(b) A set G is called a group over the binary operation $*$ such that $a, b \in G$

Binary operation $*$ is associative under addition

there exist an element $\{e\}$ such that $a*e = e*a = e$

for every element a there exist an element a' such that $a*a' = a'*a = e$.

Q3. Find all the subgroups of the group Z_{12} .

Q4. Suppose H and K are subgroups of group G . If $o(H) = 12$ and $o(K) = 35$, find $o(H \cap K)$.

[Hint: Use Lagrange's Theorem]

Q5. Suppose that K is a proper subgroup of H and H is a proper subgroup of G . If $o(K) = 42$ and $o(G) = 420$. What are possible orders of H ?

Q6. Let $o(a) = 30$. How many left cosets of $\langle a^4 \rangle$ in $\langle a \rangle$ are there?

[Hint: first find $o(a^4)$ and no of left cosets = $\text{index}[G:H]$]

Q7. Find the smallest subgroups of Z containing (i) 6 & 15 (ii) 12, 18, & 45.

[Hint: find integer k such that subgroup is $\langle k \rangle$]

Books suggested:

- 1) “A First course in Abstract Algebra” Seventh edition by John B. Fraleigh .
- 2) “Contemporary Abstract Algebra” by Joseph A Gallan.

Note: Student will have to give the detail of the references for each question.

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